1 $A = \begin{pmatrix} 2 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 0 & -1 \end{pmatrix}$

Určete, které ze součinů $A \cdot B$, $B \cdot A$, $A \cdot C$, $C \cdot A$, $B \cdot C$, $C \cdot B$ lze spočítat.

$A \cdot B$ – nelze

$B \cdot A$ – LZE

$A \cdot C$ – LZE

$C \cdot A$ – LZE

$B \cdot C$ – nelze

$C \cdot B$ – LZE

② $A = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix}$

Které ze součinů $A \cdot B$, $B^T \cdot A$, $B \cdot A$, $C \cdot B$, $B \cdot C$, $C \cdot B^T$ lze spočítat? Určete i rozměry výsledných matic.

- $A \cdot B$ – LZE

$$\underset{2 \times 2}{\begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}} \cdot \underset{2 \times 3}{\begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \end{pmatrix}} = \left(2 \times 3 \text{ matice} \right)$$

- $B^T \cdot A = \underset{3 \times 2}{\begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 1 \end{pmatrix}} \cdot \underset{2 \times 2}{\begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}} = \left(3 \times 2 \text{ matice} \right)$

LZE

- $B \cdot A$ – NELZE
- $C \cdot B$ – NELZE
- $B \cdot C = \left(2 \times 3 \text{ matice} \right)$

LZE

- $C \cdot B^T = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 1 \end{pmatrix}$ $\underline{\underline{3 \times 2}}$

③ $A = \begin{pmatrix} 3 & 0 & 2 \\ 0 & 1 & 2 \\ 2 & 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}$

Vypočtěte $(A-2I)^T \cdot B$, kde I je jednotková matice.

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow 2I = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \Rightarrow A-2I = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$$

$$(A-2I)^T \cdot B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 1 & 0 \\ 2 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 & 2 \\ 1 & 1 \\ 6 & 0 \end{pmatrix}}}$$

4 $A = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ Vypočtěte $(A-B)^2$.

$$A-B = \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix}$$

$$(A-B)^2 = (A-B)\cdot(A-B) = \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -1 & 2 \\ -4 & 7 \end{pmatrix}}}$$

5

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Ověřte, že $(A \cdot B)^T = B^T \cdot A^T$ a $A \cdot I = I \cdot A = A$

$$A \cdot B = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 5 & 1 \end{pmatrix}$$

$$(A \cdot B)^T = \underline{\underline{\begin{pmatrix} 5 & 5 \\ 2 & 1 \end{pmatrix}}}$$

$$B^T \cdot A^T = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 & 5 \\ 2 & 1 \end{pmatrix}}}$$

$$A \cdot I = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}}} = A$$

$$I \cdot A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}}} = A$$

⑥ $A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$

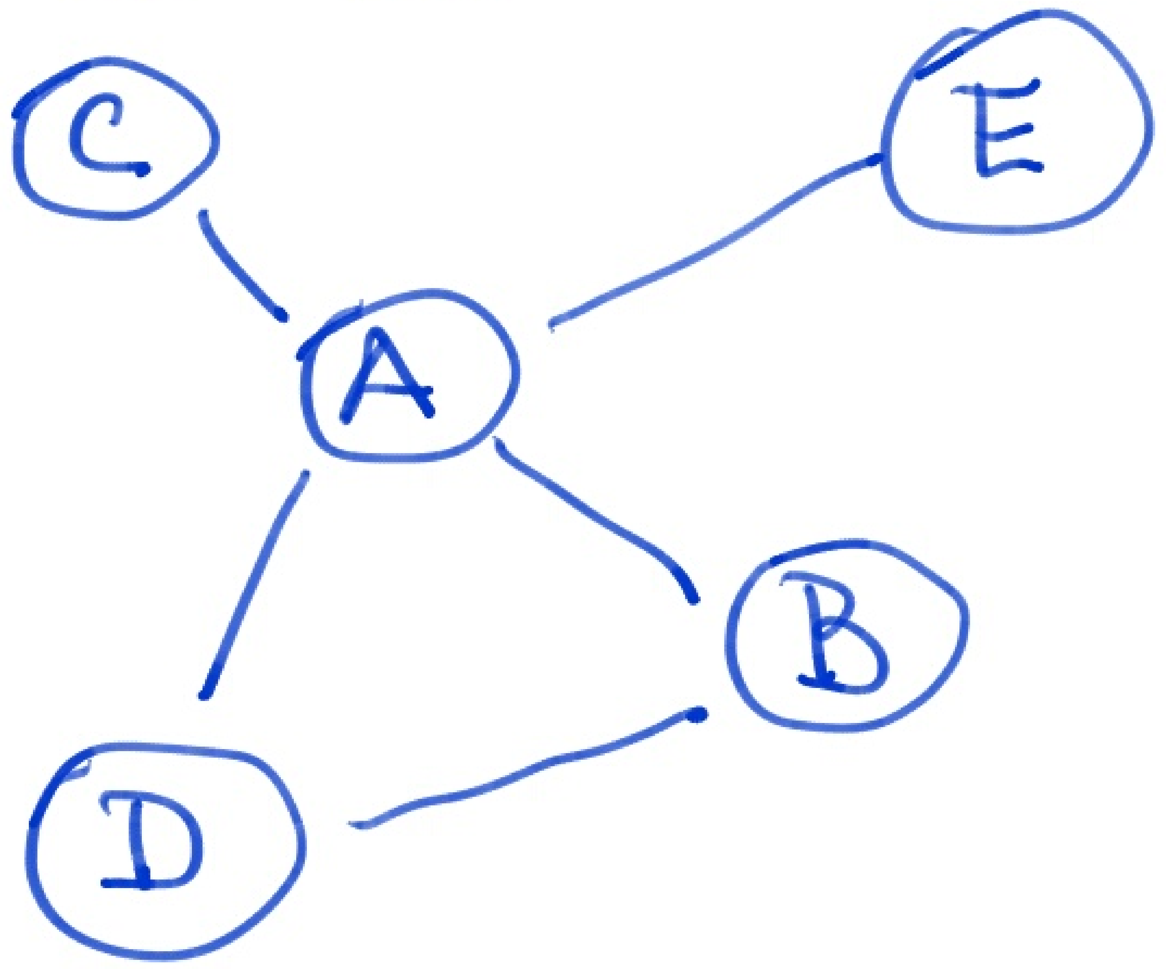
Vypočtěte $A \cdot B \cdot C$.

$$A \cdot B = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 2 & 2 \end{pmatrix}$$

$$A \cdot B \cdot C = \begin{pmatrix} 2 & 4 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 8 & 2 \\ 4 & 2 \end{pmatrix}}}$$

Lze počítat i $A \cdot (B \cdot C)$

7) Uvažujme města A, B, C, D, E a přímé letecké spoje mezi nimi, viz obrázek:



Lze zapsat do matice:

$$M = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}$$

PŘÍMÉ SPOJE

$$M^2 = M \cdot M = \begin{pmatrix} 4 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

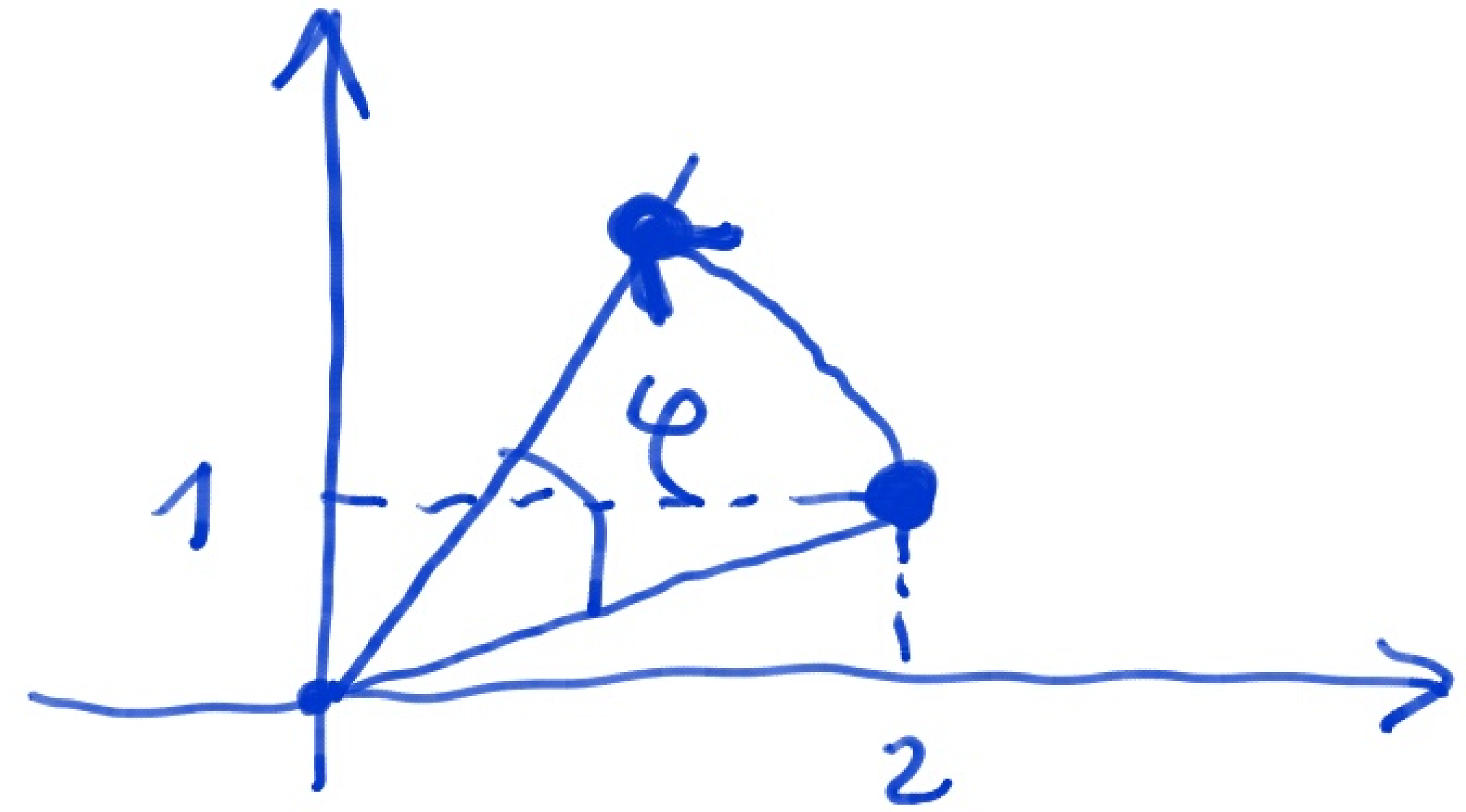
POČET TRAS S JEDNÍM PRESTUPEM

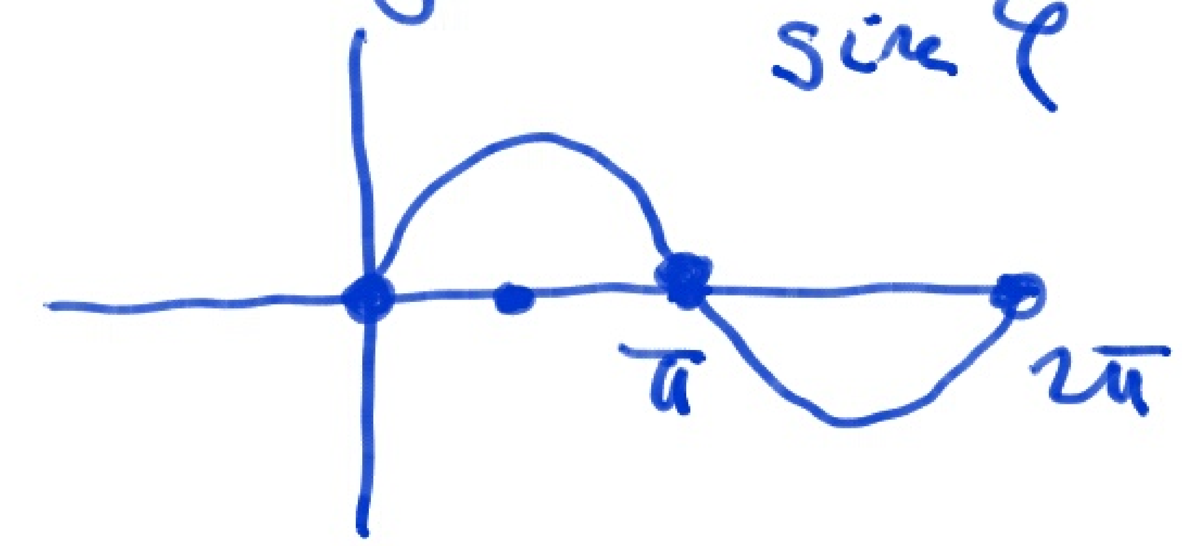
$$M + M^2 = \begin{pmatrix} 4 & 2 & 1 & 2 & 1 \\ 2 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

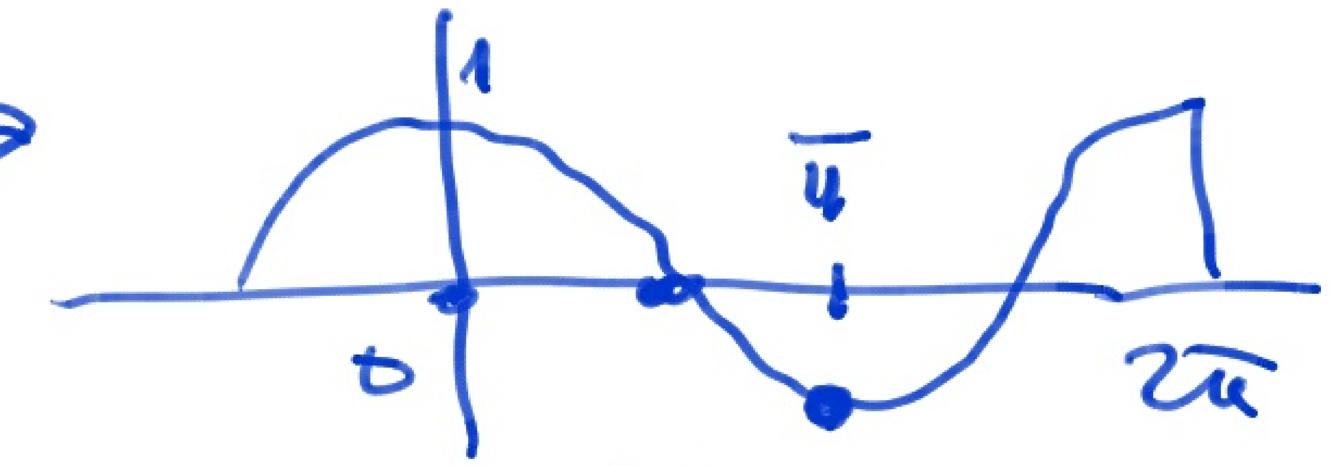
POČET SPOJŮ S MAX. JEDNÍM PRESTUPEM (zahrnuje i přímé spoje)

(8) MATICE ROTACE o úhel φ v kladném smyslu

$$R_\varphi = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

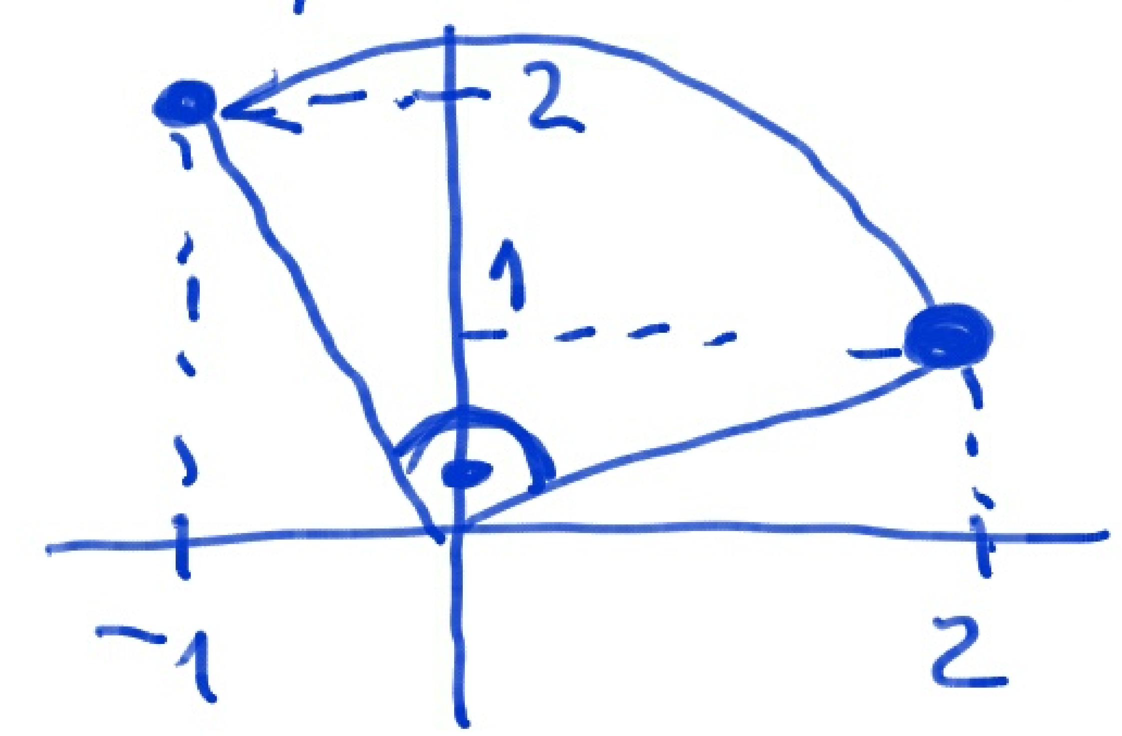






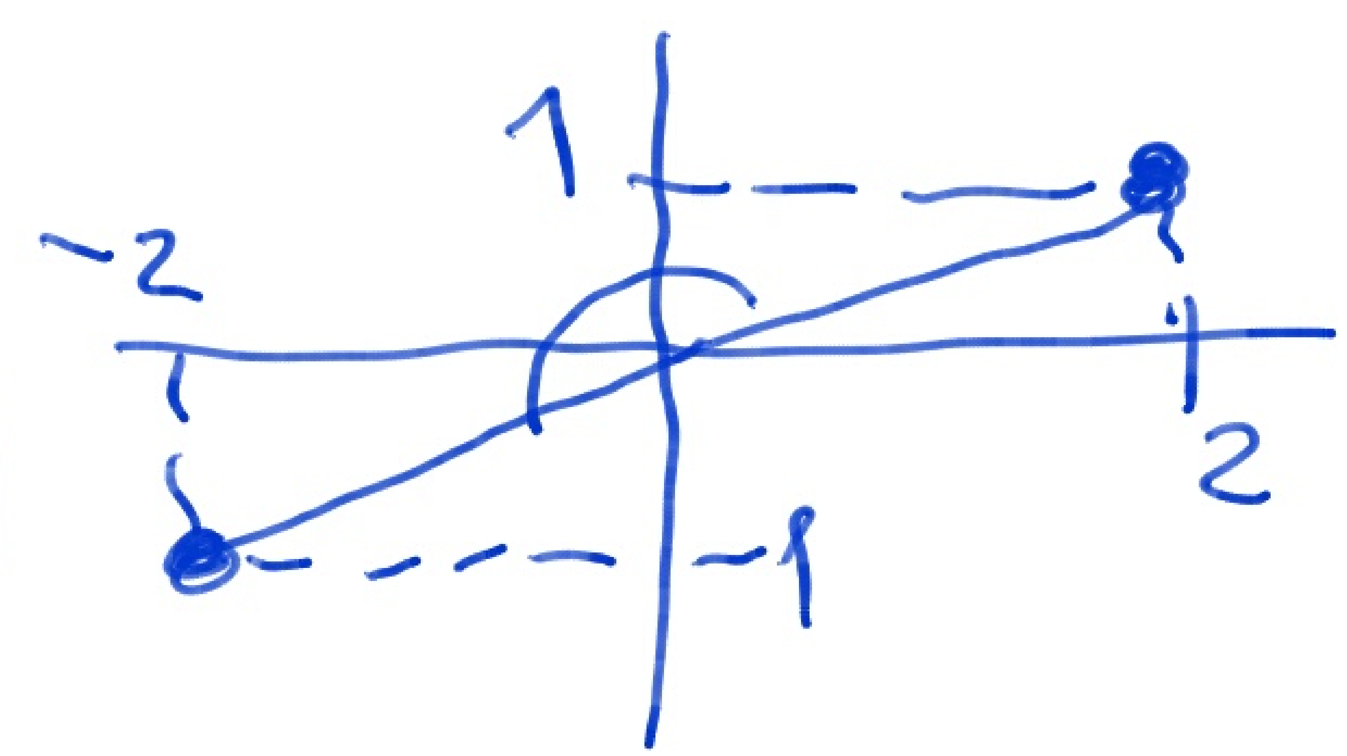
a) $\varphi = \frac{\pi}{2}$, $\sin\frac{\pi}{2} = 1$, $\cos\frac{\pi}{2} = 0$

$$\underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}_{R_{\frac{\pi}{2}}} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ 1 \end{pmatrix} \to \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

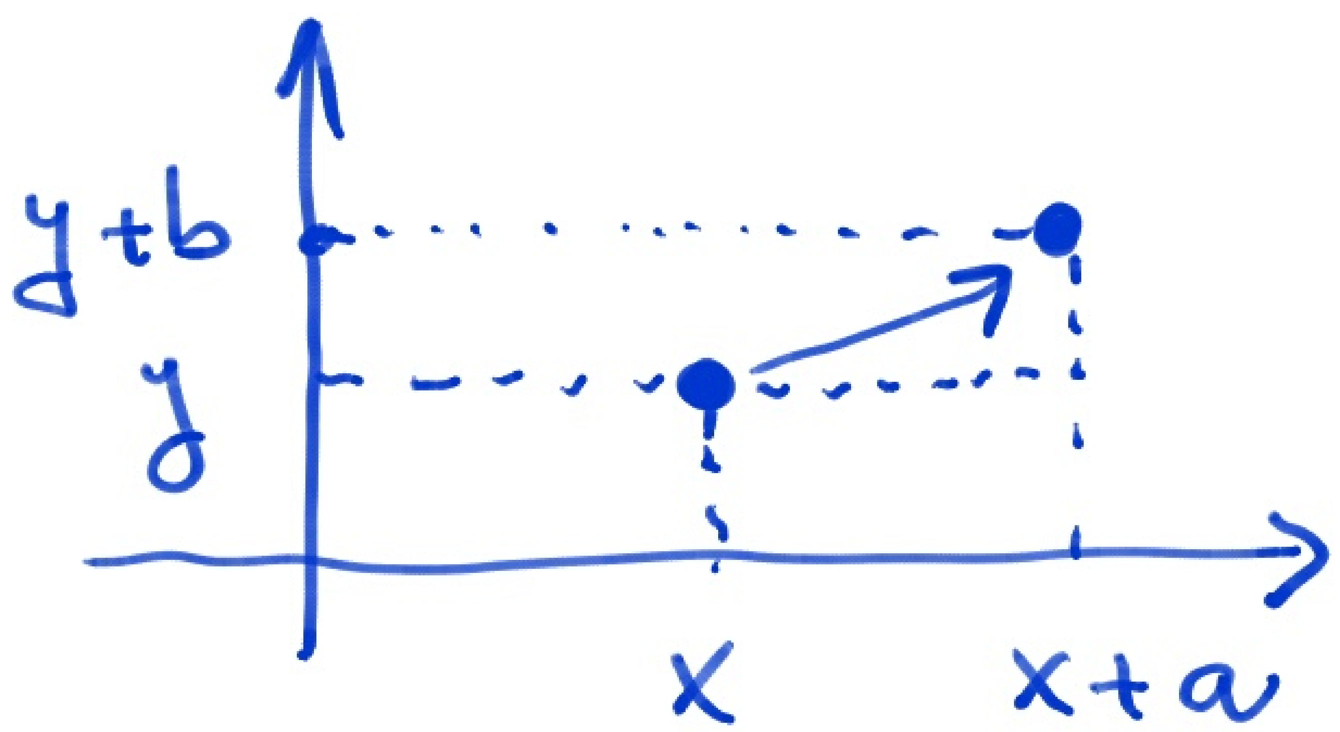


b) $\varphi = \pi$, $\sin\pi = 0$, $\cos\pi = -1$

$$\underbrace{\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}}_{R_{\pi}} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 \\ 1 \end{pmatrix} \to \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$



9 MATICE POSUNUTÍ



$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x+a \\ y+b \end{pmatrix}$$

NELZE NAJÍT 2x2 MATICI, KTERÁ BY POSUNULA POČÁTEK:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

bod $\begin{pmatrix} x \\ y \end{pmatrix}$ stotožníme s bodem $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

—"— $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ —"— ——— $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

MATICE POSUNUTÍ:

$$P_{a,b} = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x+a \\ y+b \\ 1 \end{pmatrix}$$