(A) $z = y^3 - 3xy + x^2 + x$ – lokální extrémy

$z'_x = -3y + 2x + 1 = 0$

$z'_y = 3y^2 - 3x = 0$

$y^2 - x = 0 \Rightarrow x = y^2$

$\Rightarrow -3y + 2y^2 + 1 = 0$

$2y^2 - 3y + 1 = 0$

$y_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$

$y_1 = 1 \Rightarrow x_1 = 1$

$y_2 = \frac{1}{2} \Rightarrow x_2 = \frac{1}{4}$

$\Rightarrow$ STACIONÁRNÍ BODY $[1, 1]$, $[\frac{1}{4}, \frac{1}{2}]$

$H(x,y) = \begin{vmatrix} 2 & -3 \\ -3 & 6y \end{vmatrix}$ (z''_{xx}, z''_{xy}; z''_{xy}, z''_{yy})

$\Rightarrow$ (1) $H(1,1) = \begin{vmatrix} 2 & -3 \\ -3 & 6 \end{vmatrix} = 3 > 0$ ($2 > 0$)

LOKÁLNÍ MIN

(2) $H(\frac{1}{4}, \frac{1}{2}) = \begin{vmatrix} 2 & -3 \\ -3 & 3 \end{vmatrix} = -3$

$-3 < 0$

$\Rightarrow$ NENÍ LOK. EXTRÉM

(B) $z = x^4 - 2x^2 + y^3 - 3y^2$ – LOKÁLNÍ EXTRÉMY:

$z'_x = 4x^3 - 4x = 0 \Rightarrow 4x(x^2-1) = 0 \Rightarrow x_1 = 0,\ x_2 = 1,\ x_3 = -1$

$z'_y = 3y^2 - 6y = 0 \Rightarrow 3y(y-2) = 0 \Rightarrow y_1 = 0,\ y_2 = 2$

$\Rightarrow$ STAC. BODY: $[0,0], [0,2], [1,0], [1,2], [-1,0], [-1,2]$

$$H(x,y) = \begin{vmatrix} 12x^2-4 & 0 \\ 0 & 6y-6 \end{vmatrix}$$

(1) $H(0,0) = \begin{vmatrix} -4 & 0 \\ 0 & -6 \end{vmatrix} = 24 > 0 \Rightarrow$ LOK. MAX ($-4 < 0$)

(2) $H(0,2) = \begin{vmatrix} -4 & 0 \\ 0 & 6 \end{vmatrix} = -24 < 0$

NENÍ LOK. EXTRÉM

(3) $H(1,0) = \begin{vmatrix} 8 & 0 \\ 0 & -6 \end{vmatrix} = -48 < 0$

NENÍ LOK. EXTRÉM

$$H(x,y) = \begin{vmatrix} 12x^2-4 & 0 \\ 0 & 6y-6 \end{vmatrix}$$

④ $H(1,2) = \begin{vmatrix} \textcircled{8}^{>0} & 0 \\ 0 & 6 \end{vmatrix} = 48 > 0$

LOK. MIN.

⑤ $H(-1,0) = \begin{vmatrix} 8 & 0 \\ 0 & -6 \end{vmatrix} = -48 < 0$

NENÍ LOK. EXTRÉM

⑥ $H(-1,2) = \begin{vmatrix} \textcircled{8}^{>0} & 0 \\ 0 & 6 \end{vmatrix} = 48 > 0$

LOK. MINIMUM

① $z = x^3 + xy^2 - 6xy$ – Najděte lokální extrémy

$z'_x = 3x^2 + y^2 - 6y = 0$

$z'_y = 2xy - 6x = 0$

$2x(y-3) = 0$

$\Downarrow$

1) $x = 0$

2) $y = 3$

1) $x = 0$: dosadím do 1. rce:

$y^2 - 6y = 0$

$y(y-6) = 0$

$\Downarrow$

$y = 0$, $y = 6$ $\Rightarrow$ $[0,0]$, $[0,6]$

2) $y = 3$: dosadím do 1. rce:

$3x^2 + 9 - 18 = 0$

$3x^2 = 9$

$x^2 = 3 \Rightarrow x = \pm\sqrt{3} \Rightarrow$ $[\sqrt{3}, 3]$

$[-\sqrt{3}, 3]$

STACIONÁRNÍ BODY: $[0,0]$, $[0,6]$, $[\sqrt{3},3]$, $[-\sqrt{3},3]$

$z'_x = 3x^2 + y^2 - 6y$

$z'_y = 2xy - 6x$

$z''_{xx} = 6x$

$z''_{yy} = 2x$

$z''_{xy} = 2y - 6$

$$H(x,y) = \begin{vmatrix} 6x & 2y-6 \\ 2y-6 & 2x \end{vmatrix}$$

1) $H(0,0) = \begin{vmatrix} 0 & -6 \\ -6 & 0 \end{vmatrix} = -36 < 0$ NENÍ LOK. EXTR.

2) $H(0,6) = \begin{vmatrix} 0 & 6 \\ 6 & 0 \end{vmatrix} = -36 < 0$ — " —

3) $H(\sqrt{3},3) = \begin{vmatrix} 6\sqrt{3} & 0 \\ 0 & 2\sqrt{3} \end{vmatrix} = 12 \cdot 3 > 0$ ($6\sqrt{3} > 0$)

JE LOK. EXTRÉM

=> LOK. MIN.

4) $H(-\sqrt{3},3) = \begin{vmatrix} -6\sqrt{3} & 0 \\ 0 & -2\sqrt{3} \end{vmatrix} = 12 \cdot 3 > 0$ ($-6\sqrt{3} < 0$)

JE LOK. EXTRÉM

=> LOK. MAX.

② $z = x^3 + y^2 - 6xy$ – Najděte stacionární bod

$z'_x = 3x^2 - 6y = 0 \quad /:3$

$z'_y = 2y - 6x = 0 \quad /:2$

$x^2 - 2y = 0$

$y - 3x = 0 \Rightarrow y = 3x$

dosadíme do 1. rce:

$x^2 - 6x = 0$

$x(x-6) = 0$

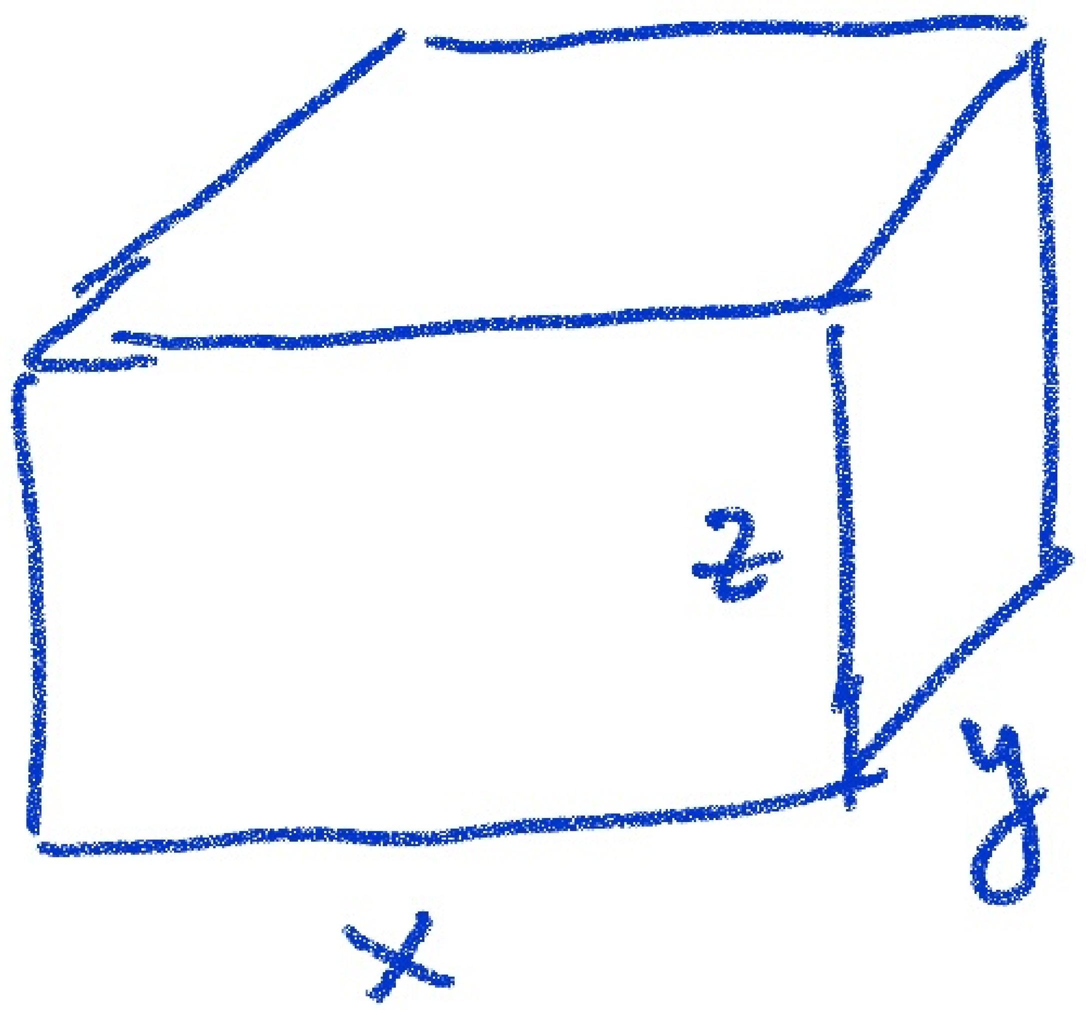
1) $x = 0$, 2) $x = 6$

1) $x = 0, y = 0$

2) $x = 6, y = 18$

$\Rightarrow [0,0], [6,18]$

③ <u>Nejlevnější bazén</u>: Určete rozměry bazénu daného objemu $\underline{V}$ s obdélníkovým dnem tak, aby se na jeho vyzdění spotřebovalo co nejméně materiálu.



$$V = x \cdot y \cdot z \quad (\text{konst.}) \implies \boxed{z = \frac{V}{xy}}$$

$$S(x,y,z) = xy + 2x\underline{z} + 2y\underline{z} \to \min$$

$$S(x,y) = xy + 2x \cdot \frac{V}{xy} + 2y \cdot \frac{V}{xy}$$

$$\boxed{S(x,y) = xy + \frac{2V}{y} + \frac{2V}{x} \to \min}$$

$x > 0, y > 0, z > 0$

$$S = xy + \frac{2V}{y} + \frac{2V}{x}, \quad x>0,\ y>0,\ z>0$$

$$\frac{2V}{x} = 2V \cdot \frac{1}{x} = 2V \cdot x^{-1}$$

$$\frac{\partial S}{\partial x} = y - 2V \cdot x^{-2} = \boxed{y - \frac{2V}{x^2} = 0}$$

$$\frac{\partial S}{\partial y} = x - 2V \cdot y^{-2} = \boxed{x - \frac{2V}{y^2} = 0}$$

$$\Rightarrow \underline{y = \frac{2V}{x^2}}$$

$$y^2 = \frac{4V^2}{x^4}$$

$$\frac{1}{y^2} = \frac{x^4}{4V^2}$$

$$x - 2V \cdot \frac{x^4}{4V^2} = 0$$

$$x - \frac{x^4}{2V} = 0$$

$$x\left(1 - \frac{x^3}{2V}\right) = 0 \Rightarrow x^3 = 2V, \quad \boxed{x = \sqrt[3]{2V}}$$

$$\Rightarrow \boxed{y} = \frac{2V}{\sqrt[3]{4V^2}} = \sqrt[3]{\frac{8V^3}{4V^2}} = \boxed{\sqrt[3]{2V}}$$

$$z = \frac{V}{xy}, \quad x = \sqrt[3]{2V}, \quad y = \sqrt[3]{2V}$$

$$z = \frac{V}{\sqrt[3]{4V^2}} = \sqrt[3]{\frac{V^3}{4V^2}} = \sqrt[3]{\frac{V}{4}}$$