$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + c} \quad \boxed{n \neq -1}$$

① $$\int x^2\,dx = \frac{x^3}{3} + c$$

$$\int x^5\,dx = \frac{x^6}{6} + c$$

② $$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + c = \underline{-\frac{1}{x} + c}$$

③ $$\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c = \underline{\underline{\frac{3}{4} x^{\frac{4}{3}} + c}}$$

④ $$\int \frac{1}{\sqrt{x}}\,dx = \int \frac{1}{x^{1/2}}\,dx = \int x^{-\frac{1}{2}}\,dx = \frac{x^{1/2}}{\frac{1}{2}} + c = \underline{\underline{2\sqrt{x} + c}}$$

⑤ $$\int \frac{1}{x}\,dx = \underline{\underline{\ln|x| + c}}$$

⑥ $\int(x^5-4x^3+2x-1)\,dx = \frac{x^6}{6}-4\cdot\frac{x^4}{4}+2\cdot\frac{x^2}{2}-x+c$

$= \frac{x^6}{6}-x^4+x^2-x+c$

⑦ $\int(x+1)\cdot x^2\,dx = \int(x^3+x^2)\,dx = \frac{x^4}{4}+\frac{x^3}{3}+c$

⑧ $\int\frac{x^2+2x-4}{x}\,dx = \int\left(\frac{x^2}{x}+\frac{2x}{x}-\frac{4}{x}\right)dx$

$= \int\left(x+2-\frac{4}{x}\right)dx = \frac{x^2}{2}+2x-4\cdot\ln|x|+c$

LINEÁRNÍ VNITŘNÍ SLOŽKA

(1) $\int \sin(5x)\,dx = -\frac{1}{5}\cos(5x) + c$

(2) $\int e^{2x+1}\,dx = \frac{1}{2}e^{2x+1} + c$

(3) $\int (3x-4)^8\,dx = \frac{(3x-4)^9}{9}\cdot\frac{1}{3} = \frac{(3x-4)^9}{27} + c$ | $\int x^8\,dx = \frac{x^9}{9}$

(4) $\int \frac{1}{x+3}\,dx = \ln|x+3| + c$

$\int \frac{1}{x}\,dx = \ln|x|$

$$\boxed{\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c}$$

1. $\int \frac{2x}{x^2-5}\,dx = \underline{\underline{\ln|x^2-5| + c}}$

2. $\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx = \underline{\underline{\frac{1}{2}\ln(x^2+4) + c}}$

3. $\int \frac{2x^2}{x^3-1}\,dx = \frac{2}{3}\int \frac{3x^2}{x^3-1}\,dx = \underline{\underline{\frac{2}{3}\ln|x^3-1| + c}}$

SUBSTITUCE

1) $\int x^2 \cdot e^{x^3+1}\, dx \quad \left| \begin{array}{l} t = x^3+1 \\ dt = 3x^2 dx \\ x^2 dx = \frac{1}{3} dt \end{array} \right| = \int e^t \cdot \frac{1}{3} dt$

$= \frac{1}{3} \int e^t dt = \frac{1}{3} e^t + c = \frac{1}{3} e^{x^3+1} + c$

2) $\int \frac{\cos x}{\sin^2 x} dx = \int \frac{1}{\sin^2 x} \cdot \cos x\, dx \quad \left| \begin{array}{l} t = \sin x \\ dt = \cos x\, dx \end{array} \right|$

$= \int \frac{1}{t^2} dt = \int t^{-2} dt = \frac{t^{-1}}{-1} + c = -\frac{1}{t} + c = -\frac{1}{\sin x} + c$

③ $$\int x \cdot \sin x^2 \, dx \quad \left| \begin{array}{l} t = x^2 \\ dt = 2x\,dx \\ x\,dx = \frac{1}{2}\,dt \end{array} \right| = \int \sin t \cdot \frac{1}{2}\,dt$$

$$= \frac{1}{2}\int \sin t \, dt = -\frac{1}{2}\cos t + c = -\frac{1}{2}\cos x^2 + c$$

④ $$\int \cos^3 x \cdot \sin x \, dx \quad \left| \begin{array}{l} t = \cos x \\ dt = -\sin x\,dx \end{array} \right| = -\int t^3 \, dt$$

$$= -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$$

$\boxed{\text{PER PARTES}}$ $\boxed{\int u v' \, dx = uv - \int u' v \, dx}$

$\underbrace{\text{POLYNOM}}_{u} \cdot \underbrace{\begin{matrix} \sin x \\ \cos x \\ e^x \end{matrix}}_{v'}$

1. $\int \underbrace{x}\cdot\underbrace{e^x}\, dx = \left| \begin{matrix} u = x & v' = e^x \\ u' = 1 & v = e^x \end{matrix} \right|$

$= x \cdot e^x - \int e^x \, dx = \underline{\underline{x \cdot e^x - e^x + c}}$

2. $\int \underbrace{x^2}\cdot\underbrace{e^x}\, dx \left| \begin{matrix} u = x^2 & v' = e^x \\ u' = 2x & v = e^x \end{matrix} \right| = x^2 \cdot e^x - \int \underbrace{2x}\cdot\underbrace{e^x}\, dx \left| \begin{matrix} u = 2x & v' = e^x \\ u' = 2 & v = e^x \end{matrix} \right|$

$= x^2 \cdot e^x - \left[2x \cdot e^x - \int 2 \cdot e^x \, dx \right] = x^2 \cdot e^x - 2x \cdot e^x + 2 \int e^x \, dx$

$= x^2 \cdot e^x - 2x \cdot e^x + 2e^x + c = \underline{\underline{e^x (x^2 - 2x + 2) + c}}$

③ $\int \underbrace{(x+2)}_{} \cdot \underbrace{\sin x}_{} \, dx \quad \left| \begin{array}{ll} u = x+2 & v' = \sin x \\ u' = 1 & v = -\cos x \end{array} \right|$

$= -(x+2) \cdot \cos x - \int -\cos x \, dx$

$= -(x+2) \cdot \cos x + \int \cos x \, dx$

$= \underline{\underline{-(x+2) \cdot \cos x + \sin x + C}}$

DRUHÝ TYP:

$\underbrace{\text{polynom}}_{v'} \cdot \underbrace{\begin{matrix}\ln x\\ \operatorname{arctg} x\end{matrix}}_{u}$

$$\boxed{\int u v' \, dx = uv - \int u' v \, dx}$$

1) $\int \underbrace{x}\cdot\underbrace{\ln x}\,dx \left| \begin{matrix} u = \ln x & v' = x \\ u' = \frac{1}{x} & v = \frac{x^2}{2} \end{matrix} \right| = \frac{x^2}{2}\cdot \ln x - \int \frac{1}{x}\cdot\frac{x^2}{2}\,dx$

$$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2} = \underline{\underline{\frac{x^2}{2}\ln x - \frac{x^2}{4} + c}}$$

(2) $$\int \arctan x \, dx = \int \underbrace{1}\cdot\underbrace{\arctan x}\, dx \quad \left| \begin{array}{ll} u = \arctan x & v' = 1 \\ u' = \dfrac{1}{x^2+1} & v = x \end{array} \right|$$

$$= x \cdot \arctan x - \int \frac{1}{x^2+1} \cdot x \, dx$$

$$\boxed{\int \frac{f'}{f} dx = \ln|f| + c}$$

$$= x \cdot \arctan x - \int \frac{x}{x^2+1} dx$$

$$= x \cdot \arctan x - \frac{1}{2} \int \frac{2x}{x^2+1} dx$$

$$= \underline{\underline{x \cdot \arctan x - \frac{1}{2} \ln(x^2+1) + c}}$$

$\int x \cdot \cos x \, dx$ ... per partes 1x

$\int x^2 \cdot \cos x \, dx$ ... per partes 2x

$\int x \cdot \cos x^2 \, dx$ ... substituce $t = x^2$