DERIVACE – VÝPOČET

$$\boxed{(x^n)' = n \cdot x^{n-1}}$$

① $f(x) = x^5 \Rightarrow f'(x) = 5x^4$

$g(x) = x^8 \Rightarrow g'(x) = 8 \cdot x^7$

② $f(x) = \frac{1}{x^3} = x^{-3}$

$$\boxed{\frac{1}{x^n} = x^{-n}}$$

$f'(x) = -3 \cdot x^{-4} = -3 \cdot \frac{1}{x^4} = -\frac{3}{x^4}$

$$\boxed{(x^n)' = n \cdot x^{n-1}}$$

$$\boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$$

③ $y = \sqrt{x} = x^{\frac{1}{2}}$

$y' = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

④ $y = \sqrt[3]{x} = x^{\frac{1}{3}}$

$y' = \frac{1}{3} \cdot x^{-\frac{2}{3}} = \frac{1}{3} \cdot \frac{1}{x^{2/3}} = \frac{1}{3 \cdot \sqrt[3]{x^2}}$

⑤ $y = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-\frac{1}{2}} \Rightarrow y' = -\frac{1}{2} \cdot x^{-\frac{3}{2}} = -\frac{1}{2} \cdot \frac{1}{x^{3/2}} = -\frac{1}{2\sqrt{x^3}}$

$$(u \pm v)' = u' \pm v' \qquad (c \cdot u)' = c \cdot u'$$

1. $y = 3x^2 \Rightarrow y' = 3 \cdot 2x = 6x$

2. $y = 3x^3 - 5x^2 + 8x - 1$

$$y' = 3 \cdot 3x^2 - 5 \cdot 2x + 8 \cdot 1 - 0$$

$$= 9x^2 - 10x + 8$$

$\boxed{\text{SOUČIN A PODÍL}}$

$$\boxed{\begin{aligned}(u\cdot v)' &= u'\cdot v + u\cdot v' \\ \left(\frac{u}{v}\right)' &= \frac{u'\cdot v - u v'}{v^2}\end{aligned}}$$

1. $y = \underbrace{x^3}_{u} \cdot \underbrace{\ln x}_{v}$

$$y' = 3\cdot x^2\cdot \ln x + x^3\cdot \frac{1}{x}$$

$$= \underline{3x^2\cdot \ln x + x^2}$$

$$(\ln x)' = \frac{1}{x}$$

$$(e^x)' = e^x$$

2. $y = \frac{e^x}{x^2}$

$$y' = \frac{e^x\cdot x^2 - e^x\cdot 2x}{x^4} = \frac{e^x\cdot x\,(x-2)}{x^4} = \underline{\underline{\frac{e^x(x-2)}{x^3}}}$$

$$\boxed{\begin{aligned}(u \cdot v)' &= u' \cdot v + u v' \\ \left(\frac{u}{v}\right)' &= \frac{u' \cdot v - u v'}{v^2}\end{aligned}}$$

(3) $y = \dfrac{x^2}{x-2}$

$$y' = \frac{2x \cdot (x-2) - x^2 \cdot 1}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - x^2}{(x-2)^2} = \frac{x^2 - 4x}{(x-2)^2}$$

$$(\sin x)' = \cos x$$

(4) $y = \sqrt{x} \cdot \sin x = x^{1/2} \cdot \sin x$

$$y' = \frac{1}{2} x^{-\frac{1}{2}} \cdot \sin x + x^{1/2} \cdot \cos x = \underline{\frac{1}{2\sqrt{x}} \cdot \sin x + \sqrt{x} \cdot \cos x}$$

SLOŽENÁ FUNKCE

$$(g(f(x)))' = g'(f(x)) \cdot f'(x)$$

$$(\cos \underbrace{x}_{x^2})' = -\sin \underbrace{x}_{x^2}$$

1. $y = \cos x^2$

$$y' = -\sin \underbrace{x^2} \cdot 2\underbrace{x} = -2x \cdot \sin x^2$$

$$((\underbrace{X}_{\cos x})^2)' = 2\underbrace{X}_{\cos x}$$

2. $y = \cos^2 x = (\underbrace{\cos x})^2$

$$y' = 2 \cdot \cos x \cdot (-\sin x) = -2 \cos x \cdot \sin x$$

③ $y = e^{x^2+1}$ $\qquad (e^{x})' = e^{x}$

$$\underline{y' = e^{x^2+1} \cdot 2x}$$

④ $y = (x^3-2x+1)^5$ $\qquad (x^5)' = 5x^4$

$$y' = 5 \cdot (x^3-2x+1)^4 \cdot (3x^2-2)$$

⑤ $y = e^{\sin 5x} \Rightarrow \underbrace{y' = e^{\sin 5x} \cdot \cos 5x \cdot 5}$

6 $y = \ln \frac{x-1}{x+1}$

$(\ln x)' = \frac{1}{x}$

$$y' = \frac{x+1}{x-1} \cdot \frac{1 \cdot (x+1) - (x-1) \cdot 1}{(x+1)^2}$$

$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$

$$= \frac{x+1-x+1}{(x-1) \cdot (x+1)}$$

$$= \frac{2}{(x-1)(x+1)} = \underline{\underline{\frac{2}{x^2-1}}}$$

PARCIÁLNÍ DERIVACE

(1) $z = x^3y^2 - 4xy^2 + 5x^2y - 2x^2 - 4$

$z'_x = 3x^2 \cdot y^2 - 4 \cdot 1 \cdot y^2 - 5 \cdot 2x \cdot y - 4x$

$= 3x^2y^2 - 4y^2 - 10xy - 4x$

$z'_y = x^3 \cdot 2y - 4x \cdot 2y + 5x^2 \cdot 1$

$= 2x^3y - 8xy + 5x^2$

(2) $z = \underbrace{xy^2}\cdot\underbrace{e^{x+y}}$ $\qquad (u\cdot v)' = u'\cdot v + u\cdot v'$

$$z'_x = y^2\cdot e^{x+y} + xy^2\cdot e^{x+y}$$
$$= y^2\cdot e^{x+y}(1+x)$$

$$z'_y = x\cdot 2y\cdot e^{x+y} + xy^2\cdot e^{x+y}$$
$$= xy\cdot e^{x+y}(2+y)$$

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$$z = \frac{x}{y} = x \cdot y^{-1}$$

$$z'_x = 1 \cdot y^{-1} = y^{-1} = \frac{1}{y}$$

$$z'_y = x \cdot (-1) \cdot y^{-2} = -\frac{x}{y^2}$$

JAKO PODÍL:

$$z'_x = \frac{1 \cdot y - x \cdot 0}{y^2} = \frac{1}{y}$$

$$z'_y = \frac{0 \cdot y - x \cdot 1}{y^2} = \frac{-x}{y^2}$$

DERIVACE VYŠŠÍCH ŘÁDŮ

(1) $y = 3x^3 - 5x^2 + 2x + 7$

$y' = 9x^2 - 10x + 2$

$y'' = 18x - 10$

$y''' = 18$

$y^{(4)} = 0$

$y^{(5)} = 0$

$\vdots$

(2)

$$z = 3x^2y^2 + 4xy^2 - 3x$$

$$z'_x = 6xy^2 + 4y^2 - 3$$

$$z'_y = 6x^2y + 8xy$$

$$z''_{xx} = 6y^2$$

$$z''_{yx} = 12xy + 8y$$

$$z''_{xy} = 12xy + 8y$$

$$z''_{yy} = 6x^2 + 8x$$