INTEGRÁL, PRIMITIVNÍ FUNKCE

$$\begin{tikzcd} f(x) \arrow[r, bend left, "\text{int.}"] & F(x) \arrow[l, bend left, "\text{deriv.}"] \end{tikzcd}$$

$$F'(x) = f(x)$$

$f(x)$ je derivace funkce $F(x)$

$F(x)$ je primitivní funkce k $f(x)$

$$\int f(x)\,dx = F(x) + c$$

$f(x) = 2x \qquad F(x) = x^2$

$$(x^2)' = 2x$$
$$(x^2 + 1)' = 2x$$
$$(x^2 + 5)' = 2x$$
$$\int 2x\,dx = x^2 + c$$

Primitivních funkcí k dané funkci je ∞, liší se o přičtenou konstantu.

Mocninné funkce

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Vzorec platí pro každé $n \in \mathbb{R}$

kromě $n = -1$

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + c$$

$$\int x^3 dx = \frac{x^4}{4} + c$$

$$\int x^5 dx = \frac{x^6}{6} + c$$

$$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} = -\frac{1}{x} + c$$

$$\int \sqrt{x}\, dx = \int x^{\frac{1}{2}} dx = \frac{x^{3/2}}{\frac{3}{2}} + c = \frac{2}{3} x^{3/2} + c = \frac{2}{3}\sqrt{x^3} + c$$

$$\int \frac{1}{\sqrt{x}} dx = \int \frac{1}{x^{1/2}} dx = \int x^{-1/2} dx = \frac{x^{1/2}}{1/2} + c = 2x^{1/2} + c = 2\sqrt{x} + c$$

Součet, násobek s konstantou, polynom

$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int c \cdot f(x)\,dx = c \cdot \int f(x)\,dx$$

$$\int (x^3 + 2x - 1)\,dx = \frac{x^4}{4} + 2 \cdot \frac{x^2}{2} - x + c = \underline{\frac{x^4}{4} + x^2 - x + c}$$

$$\int (x^4 + 2x^3 - 5)\,dx = \frac{x^5}{5} + 2 \cdot \frac{x^4}{4} - 5x + c = \underline{\frac{x^5}{5} + \frac{x^4}{2} - 5x + c}$$

součin, podíl funkcí
+ složené funkce:
nemáme žádné obecné pravidla

$$\boxed{\int f(\underline{x})\,dx = F(x) \implies \int f(\underline{ax+b})\,dx = \frac{1}{a}F(\underline{ax+b})}$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \cos(2x)\,dx = \frac{1}{2}\sin(2x) + c \Leftarrow (\sin(2x))' = \cos(2x)\cdot 2$$

$$\int \cos(3x+2)\,dx = \frac{1}{3}\sin(3x+2) + c$$

$$\int e^x\,dx = e^x + c$$

$$\int e^{5x-1}\,dx = \frac{1}{5}e^{5x-1} + c$$

$$\int e^{-x}\,dx = -e^{-x} + c$$

$$\int (2x+3)^3\,dx = \frac{1}{2}\cdot\frac{(2x+3)^4}{4} + c = \frac{(2x+3)^4}{8} + c$$

$$\left(\int x^3\,dx = \frac{x^4}{4} + c\right)$$

$$\int \frac{1}{x+2}\,dx = \underline{\ln|x+2| + c}$$

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

$$\boxed{\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c}$$

spec. $\int \frac{1}{x}\,dx = \ln|x| + c$

$$\int \frac{1}{x+2}\,dx = \ln|x+2| + c$$

$$\int \frac{2x}{x^2+3}\,dx = \ln(x^2+3) + c$$

$$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \frac{1}{2}\ln(x^2+1) + c$$

$$\int \operatorname{tg} x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{-\sin x}{\cos x}\,dx = -\ln|\cos x| + c$$