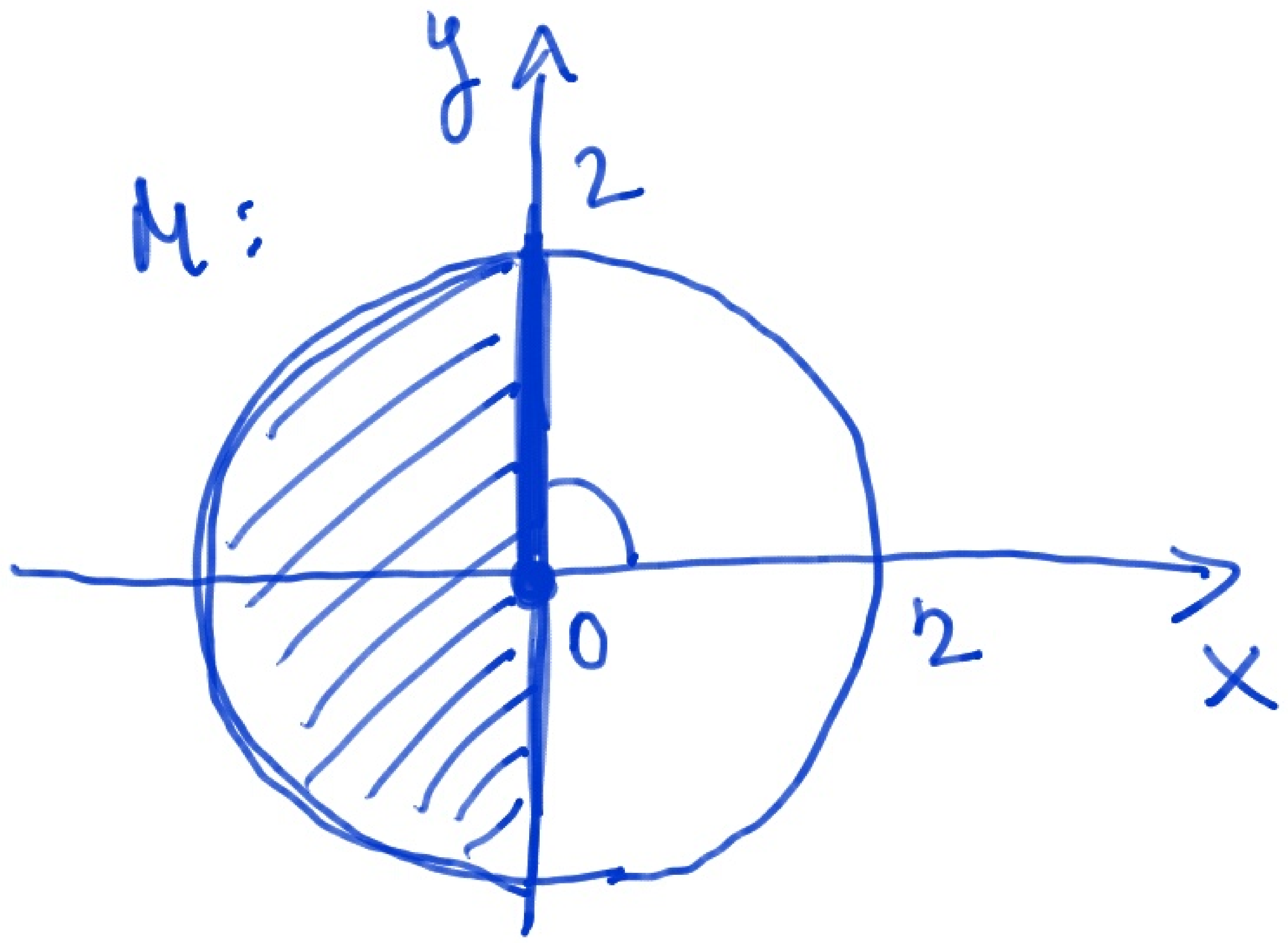
① $\iint\limits_M (x^2+y^2-y)\,dx\,dy$, $M:\ x^2+y^2 \le 4,\ x \le 0$

M:



$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[\int_0^2 (r^2 - r\cdot\sin\varphi)\cdot r\,dr \right] d\varphi$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[\int_0^2 (r^3 - r^2\cdot\sin\varphi)\,dr \right] d\varphi$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[\frac{r^4}{4} - \frac{r^3}{3}\sin\varphi \right]_0^2 = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(4 - \frac{8}{3}\sin\varphi\right) d\varphi$$

$$= \left[4\varphi + \frac{8}{3}\cos\varphi \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= 6\pi + \frac{8}{3}\cdot\underbrace{\cos\frac{3\pi}{2}}_{=0} - 2\pi - \frac{8}{3}\cdot\underbrace{\cos\frac{\pi}{2}}_{=0} = \underline{\underline{4\pi}}$$

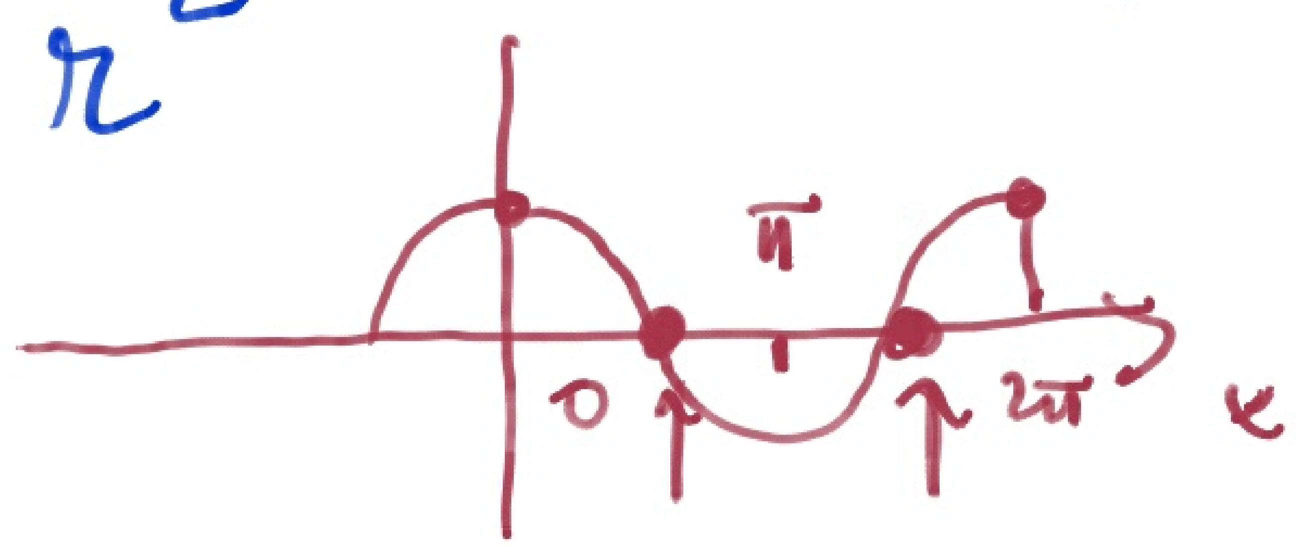
$x = r\cdot\cos\varphi$

$y = r\cdot\sin\varphi$

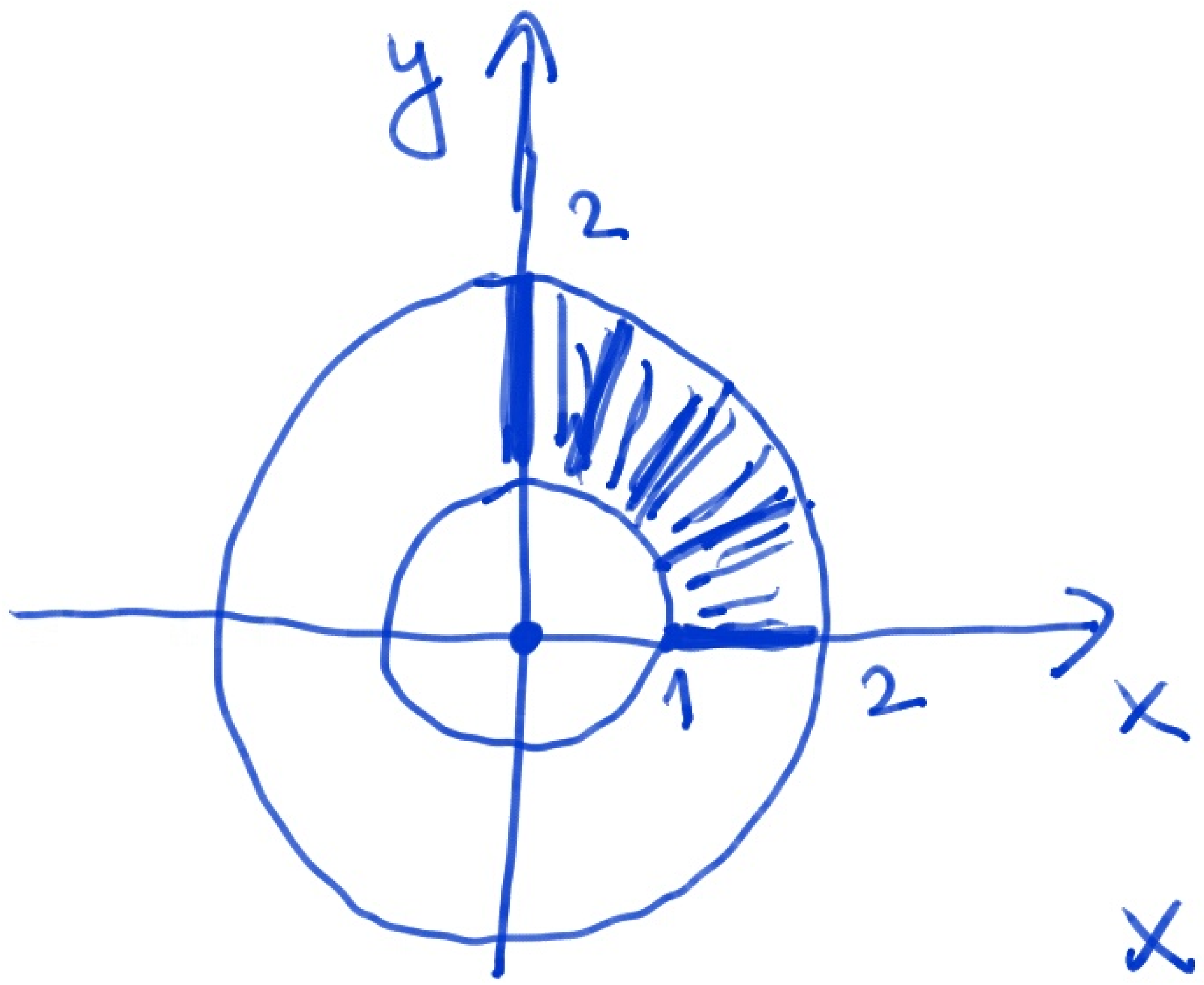
$x^2+y^2 = r^2$

$\frac{\pi}{2} \le \varphi \le \frac{3\pi}{2}$

$0 \le r \le 2$



(2) $\iint_M x\sqrt{x^2+y^2}\,dx\,dy \qquad x^2+y^2 \geq 1, \quad x^2+y^2 \leq 4, \quad x \geq 0, \; y \geq 0$



$$\int_0^{\frac{\pi}{2}}\left[\int_1^2 \underbrace{r\cdot\cos\varphi\cdot\sqrt{r^2}\cdot r}_{r^3\cdot\cos\varphi}\,dr\right]d\varphi$$

$$= \int_0^{\frac{\pi}{2}}\cos\varphi\,d\varphi \cdot \int_1^2 r^3\,dr$$

$x = r\cdot\cos\varphi$

$x^2+y^2 = r^2$

$0 \leq \varphi \leq \frac{\pi}{2}$

$1 \leq r \leq 2$

$$= \left[\sin\varphi\right]_0^{\frac{\pi}{2}} \cdot \left[\frac{r^4}{4}\right]_1^2$$

$$= \Big(\underbrace{\sin\frac{\pi}{2}}_{1} - \underbrace{\sin 0}_{0}\Big)\cdot\left(4-\frac{1}{4}\right) = \frac{15}{4}$$