Sample exam – solution

① $\left(\begin{array}{cccc|c} 1 & 2 & 0 & -1 & -2 \\ 2 & 3 & 1 & -5 & 1 \\ 1 & 1 & 1 & -4 & 3 \\ 0 & 1 & -1 & 2 & 0 \end{array}\right) \begin{array}{l} /\cdot(-2)\ /\cdot(-1) \\ + \\ + \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & -1 & -2 \\ 0 & -1 & 1 & -3 & 5 \\ \cancel{0} & \cancel{-1} & \cancel{1} & \cancel{-3} & \cancel{5} \\ 0 & 1 & -1 & 2 & 0 \end{array}\right) + \sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & -1 & -2 \\ 0 & -1 & 1 & -3 & 5 \\ 0 & 0 & 0 & -1 & 5 \end{array}\right)$

$\underline{\underline{x_4 = -5}}$ | $-x_2 + x_3 + 15 = 5$ | $\underline{\underline{x_3 = t}}$ | $x_1 + 2t + 20 + 5 = -2$

$\underline{\underline{x_2 = t + 10}}$ $\qquad$ $\underline{\underline{x_1 = -2t - 27}}$

② a) $A^T - I = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & 2 \\ 3 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 \\ 1 & 1 & 2 \\ 3 & 1 & -1 \end{pmatrix}$

$(A^T - I) \cdot A = \begin{pmatrix} 0 & 2 & 2 \\ 1 & 1 & 2 \\ 3 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 3 \\ 2 & 2 & 1 \\ 2 & 2 & 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 8 & 8 & 2 \\ 7 & 7 & 4 \\ 3 & 3 & 10 \end{pmatrix}}}$

b) $\det A = \begin{vmatrix} 1 & 1 & 3 \\ 2 & 2 & 1 \\ 2 & 2 & 0 \end{vmatrix} = 0 + 12 + 2 - 12 - 2 - 0 = \underline{\underline{0}}$

$\begin{matrix} 1 & 1 & 3 \\ 2 & 2 & 1 \end{matrix}$

c) The rows are lin. dependent.

A^{-1} does not exist.

③ $A \cdot A^{-1} = I = A^{-1} \cdot A$

$(A \mid I) \sim$ row operations $\sim (I \mid A^{-1})$

④ a) $\displaystyle\int \frac{x^3 - x + 1}{x}\,dx = \int \left(x^2 - 1 + \frac{1}{x}\right) dx = \underline{\underline{\frac{x^3}{3} - x + \ln|x| + c}}$

b) $\displaystyle\int x \cdot \sin x^2\,dx \quad \left| \begin{array}{l} t = x^2 \\ dt = 2x\,dx \end{array} \right| = \frac{1}{2} \int \sin t\,dt = -\frac{1}{2}\cos t + c$

$\displaystyle = \underline{\underline{-\frac{1}{2}\cos x^2 + c}}$

c) $\displaystyle\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + c = \underline{\underline{-\frac{1}{2x^2} + c}}$

5 a) $\int_a^b f(x)\,dx = F(b) - F(a)$, $F' = f$.

b) $\int_0^1 (x^2-1)\,dx = \left[\frac{x^3}{3} - x\right]_0^1 = \frac{1}{3} - 1 = -\frac{2}{3}$

6 a) $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

b) one-to-one : $y = x^3$

not one-to one : $y = x^2$

c) $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$

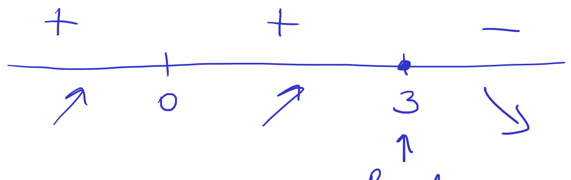
7 a) $y' = \frac{1}{2\sqrt{x}}(x-5) + \sqrt{x} = \frac{1}{2}\sqrt{x} - \frac{5}{2\sqrt{x}} + \sqrt{x} = \frac{3\sqrt{x}}{2} - \frac{5}{2\sqrt{x}}$

b) $y' = 2x \cdot \cos x - x^2 \cdot \sin x$

c) $y' = \frac{(1+\frac{1}{x})\cdot(x^2+1) - (x+\ln x)\cdot 2x}{(x^2+1)^2}$

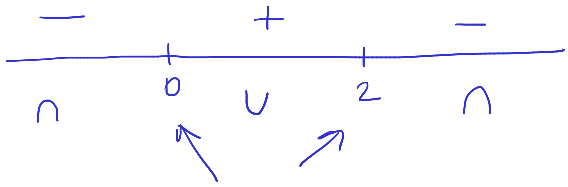
d) $y' = 3(x+\sin x^2)^2 \cdot (1 + \cos x^2 \cdot 2x)$

8 a) $y' = 12x^2 - 4x^3 = 4x^2(3-x)$



local max.

b) $y'' = 24x - 12x^2 = 12x(2-x)$



points of inflection