

HOMEWORK 2 - SOLUTION

① $A \cdot \bar{A}^{-1} = \bar{A}^{-1} \cdot A = I$

② a)
$$\begin{array}{c} \left| \begin{array}{ccc} 1 & 3 & 2 \\ 2 & 5 & 3 \\ 1 & 2 & 1 \end{array} \right| \\ \hline \begin{array}{ccc} 1 & 3 & 2 \\ 2 & 5 & 3 \end{array} \end{array} = 5 + 8 + 9 - (10 + 6 + 6) = 0 =$$

b) The rows are dependent.
 $\text{rank } A = 2$
 \bar{A}^{-1} does not exist.

③ a)
$$\begin{array}{c} \left| \begin{array}{ccc} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{array} \right| \\ \hline \begin{array}{ccc} 1 & 3 & 2 \\ 1 & 2 & 1 \end{array} \end{array} = 0 + 2 + 0 - (0 + 1 + 0) = 1 =$$

b) The rows are independent.
 $\text{rank } B = 3$
 \bar{A}^{-1} exists.

④ a)
$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 1 & 5 & -1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix}$$

b)
$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 1 & 3 & 1 & 5 \\ 1 & 5 & -1 & 3 \end{array} \right) \begin{array}{l} \leftarrow + \\ \leftarrow + \\ \leftarrow + \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 2 & -1 & 1 \\ 0 & 4 & -3 & -1 \end{array} \right) \begin{array}{l} \leftarrow + \\ \leftarrow + \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & -1 & -3 \end{array} \right)$$

$$\begin{array}{l} \underline{\underline{x_3 = 3}} \quad , \quad 2x_2 - 3 = 1 \quad , \quad x_1 + 2 + 6 = 4 \\ \quad \quad \quad 2x_2 = 4 \quad \quad \quad \underline{\underline{x_1 = -4}} \\ \quad \quad \quad \underline{\underline{x_2 = 2}} \end{array}$$