

Homework 1 - solution

① a) $C^2 = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ 6 & 10 \end{pmatrix}$

b) $C - 2I = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix} \Rightarrow (C - 2I)^T = \begin{pmatrix} -1 & 2 \\ 3 & 0 \end{pmatrix}$

$$B \cdot (C - 2I)^T = \begin{pmatrix} 1 & 3 \\ -2 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} -1 & 2 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 8 & 2 \\ 5 & -4 \\ 5 & 2 \end{pmatrix}$$

c) $A \cdot B$ - YES, $B \cdot A$ - NO, $C \cdot B^T$ - YES, $A \cdot B^T$ - NO, $C^T A$ - NO

② Rank of a matrix is a maximal number of the linear independent rows.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

$$\text{rank } A = 1$$

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 3 \end{pmatrix}$$

$$\text{rank } B = 2$$

③ a) Dependent, because $\vec{b} = 2\vec{a}$.

b) Dependent, because 3 vectors of dimension 2.

c) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{pmatrix} \begin{matrix} | (-) / (-3) \\ \leftarrow + \\ \leftarrow + \end{matrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & -3 & -2 \end{pmatrix} \Rightarrow \text{rank} = 3 \Rightarrow \text{INDEPENDENT}$