1. Consider the matrices

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 0 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 \\ -2 & 1 \\ 1 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}.$$

   (a) Calculate C^2

   (b) Calculate $B(C - 2I)^T$, where I is the identity matrix.

   (c) Consider the following products:

   $$AB, \ BA, \ CB^T, \ AB^T, \ C^T A.$$

   Decide which of them can be calculated and explain why the other ones can not be calculated.

2. Write the definition of the rank of a matrix. Give an example of a 3×3 matrix such that

   (a) the rank is 1,

   (b) the rank is 2.

3. Decide whether the following vectors are linearly dependent or independent:

   (a) $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \vec{b} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}, \vec{c} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$

   (b) $\vec{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \vec{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \vec{c} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}.$

   (c) $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \vec{c} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}.$

Instructions for writing homework:

- Write your homework with solution (not only the results).

- Take a photo of the homework and convert the picture to PDF (use https://tools.pdf24.org/en/jpg-to-pdf).

- Compress the file if it is large (use https://tools.pdf24.org/en/compress-pdf).

- Send the final PDF file to the teacher (either e-mail: fisnarov@mendelu.cz or chat in MS Teams).