Monotonicity and concavity

1. Investigate the following polynomials:
   (a) Find the domain, find the parts above/below x-axis.
   (b) Monotonicity.
   (c) Concavity.
   (d) Sketch the graph.

(a) $y = 6x^3 - 3x^6$.
(b) $y = x^5 - 5x^4$.
(c) $y = 3x - x^3$.
(d) $y = x^4 + 2x^3$.
(e) $y = 4x^3 - x^4$.
(f) $y = x^3 - 2x^2 + x$.
(g) $y = x^3 - 4x^2 + 4x$.
(h) $y = x^3 - 6x^2 + 9x$.
(i) $y = 2x^3 - 9x^2 + 12x$.

2. Investigate the following functions:
   (a) Find the domain, find the parts above/below x-axis.
   (b) Monotonicity.
   (c) Concavity.

(a) $y = \dfrac{x^2}{x-2}$
Hint: $y' = \dfrac{x^2-4x}{(x-2)^2}$, $y'' = \dfrac{8}{(x-2)^3}$.

(b) $y = \dfrac{x^2}{x+1}$
Hint: $y' = \dfrac{x^2+2x}{(x+1)^2}$, $y'' = \dfrac{2}{(x+1)^3}$.

(c) $y = \dfrac{x}{(x-2)^2}$
Hint: $y' = \dfrac{-x-2}{(x-2)^3}$, $y'' = \dfrac{2x+8}{(x-2)^4}$.

(d) $y = \dfrac{x}{(x+3)^2}$
Hint: $y' = \dfrac{3-x}{(x+3)^3}$, $y'' = \dfrac{2x-12}{(x+3)^4}$.

(e) $y = \dfrac{x^2}{(x+1)^2}$.
Hint: $y' = \dfrac{2x}{(x+1)^3}$, $y'' = \dfrac{2-4x}{(x+1)^4}$.

(f) $y = \dfrac{x}{x^2+1}$.
Hint: $y' = \dfrac{1-x^2}{(x^2+1)^2}$, $y'' = \dfrac{2x^3-6x}{(x^2+1)^3}$.

(g) $y = \dfrac{x}{(x-1)^2}$.
Hint: $y' = \dfrac{-x-1}{(x-1)^3}$, $y'' = \dfrac{2x+4}{(x-1)^4}$.

(h) $y = \dfrac{x-2}{(x-1)^2}$.
Hint: $y' = \dfrac{3-x}{(x-1)^3}$, $y'' = \dfrac{2x-8}{(x-1)^4}$.

(i) $y = x + \dfrac{1}{x+1}$.
Hint: $y' = \dfrac{x^2+2x}{(x+1)^2}$, $y'' = \dfrac{2}{(x+1)^3}$.

(j) $y = \dfrac{x^2-1}{x^3}$.
Hint: $y' = \dfrac{3-x^2}{x^4}$, $y'' = \dfrac{2x^2-12}{x^5}$.