1	2	3	4	5	6	7	8	$\sum$

Name: .....

Date: .....

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1. **[6p]** Solve the system

$$\begin{aligned} x_1 + 2x_2 \quad\quad - \quad x_4 &= -2 \\ 2x_1 + 3x_2 + x_3 - 5x_4 &= 1 \\ x_1 + \quad x_2 + x_3 - 4x_4 &= 3 \\ \quad\quad x_2 - x_3 + 2x_4 &= 0 \end{aligned}$$

2. **[8p]** Let

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 2 & 1 \\ 2 & 2 & 0 \end{pmatrix}.$$

   (a) Calculate $(A^T - I) \cdot A$, where I is the identity matrix.

   (b) Calculate $\det A$.

   (c) Using the value of $\det A$ answer the questions:

      i. Are the rows of A linearly dependent or independent?

      ii. Does the inverse matrix A^{-1} exist?

3. **[4p]** Write the definition of the inverse matrix and explain the method of finding the inverse matrix.

4. **[8p]** Find the integrals

   (a) $\displaystyle\int \frac{x^3 - x + 1}{x}\,\mathrm{d}x$

   (b) $\displaystyle\int x \sin x^2\,\mathrm{d}x$

   (c) $\displaystyle\int \frac{1}{x^3}\,\mathrm{d}x$

5. **[4p]**

   (a) Write the Newton-Leibniz formula for evaluating definite integrals.

   (b) Evaluate $\displaystyle\int_0^1 (x^2 - 1)\,\mathrm{d}x$

6. **[6p]**

   (a) Write the definition of one-to-one function.

   (b) Give an example of one-to-one function and give and example of a function which is not one-to-one.

   (c) Write the definition of the derivative of a function at x_0.

7. **[8p]** Find derivatives of the following functions.

   (a) $y = \sqrt{x}(x - 5)$

   (b) $y = x^2 \cos x$

   (c) $y = \dfrac{x + \ln x}{x^2 + 1}$

   (d) $y = (x + \sin x^2)^3$

8. **[6p]** For the function $y = 4x^3 - x^4$

   (a) find intervals, where the function is increasing and decreasing and find local extrema,

   (b) find intervals, where the function is concave up and concave down a find points of inflection.

- Passing is 25 points (including bonus points).
- Write only important things in theoretical problems, no long stories!